10th tutorial

1. Two Fences. Back to our story about apple trees and fences. Imagine we don't require all the apple trees are within a *single* fence – let's say we'll allow two fenced areas. Devise an algorithm which uses as little fence as possible, each tree is within a fenced area, and there are at most 2 fenced areas. (Aim for complexity $\mathcal{O}(n^3 \log n)$; $\mathcal{O}(n^3)$ is not much harder, and $\mathcal{O}(n^2 \log n)$ is possible but fairly hard.)

2. Longest Horizontal. Devise an algorithm which finds the longest horizontal line segment contained in a (not necessarily convex) polygon.

3. Rectangles. You are given a set of n axis-parallel rectangles; beware, they are allowed to overlap. Compute the total surface of their union.

Definition (*Reduction*): A problem L reduces to a problem M if there exists a polytime function $f : \{0,1\}^* \to \{0,1\}^*$ such that $L(x) = M(f(x))$ for every $x \in \{0,1\}^*$. Intuitively: $L \to M$ if there's a polytime algorithm reducing YES-instances of L to YES-instances of M, and NO-instances to NO-instances.

4. Properties of $\to$. Prove that the "$\to$" relation in the space of problems has the following properties:

- It is *reflexive*: $A \to A$
- It is *transitive*: $A \to B \wedge B \to C \Rightarrow A \to C$
- It is not *antisymmetric* (recall the definition)
- There exist mutually irreducible problems (A does not reduce to B and B does not reduce to A)

5. Easy Problems. Show that the following problems are polynomial-time solvable:

- 2-COLORING (decide whether a graph G can be colored with 2 colors.)
- Does G have a clique of size 42?
- Given a formula in disjunctive normal form (DNF), does it have a satisfying assignment? (An example of a formula in DNF is $(x_1 \wedge \neg x_2 \wedge x_3) \vee (x_2 \wedge x_4 \wedge x_5) \vee (\neg x_1 \wedge \neg x_2 \wedge x_4)$ – notice that it is composed of *clauses* which are a conjunction of literals, and the formula is obtained by joining the clauses with disjunctions.)