7th tutorial

Domino covering. Say we have a chessboard with some cells cut out. We would like to cover the remaining cells with 1×2 pieces such that each cell is covered by exactly one piece. It is allowed to rotate the pieces. Design an algorithm solving this problem quickly.

Marmots. There are n marmots on a meadow with m holes (everything given as integer points in the plane). When an eagle shows up, a marmot manages to run at most d meters before it is eaten by the eagle. How many marmots can save themselves if each hole can take up at most one marmot? What if it can take up up to k marmots?

Minimum Isolation. We are given an integer cuboid (a 3D solid with six flat faces and all angles being right), and some k of its unit cells are marked as dangerous. Design an algorithm which selects a subset M of cells of the cuboid such that each dangerous cell is in M, and the overall surface of M is minimized; the surface is the number of faces of the unit cells in M which are not neighbors with another cell in M).

Ford-Fulkerson and Matchings. We have translated the MAXIMUM MATCHING problem on bipartite graphs to a MAX FLOW computation with unit capacities, and seen that this can be solved in $\mathcal{O}(nm)$ by Ford-Fulkerson. Interpret what Ford-Fulkerson does on this problem – what does each step look like? I.e., design a direct algorithm solving BIPARTITE MAX MATCHING in time $\mathcal{O}(nm)$ without calling FF.

Goldberg and Heights. What would happen, if we placed the source at height $n-1$, $n-2$ or even $n-3$? Figure out which property (it terminates, gives max flow always, etc.) depends on the height of the source, and what could go wrong.