6th tutorial

Min Vertex Cover: Design an algorithm computing the smallest *vertex cover* $C \subseteq V$ of a bipartite graph G. A subset C is a vertex cover if $\forall e \in E(G) \exists v \in C : v \in e$.

Vertex Connectivity: An undirected graph G has *edge connectivity* k if removing any set of $k-1$ edges does not disconnect the graph, but there exists a subset of edges F of size k such that $(V, E \setminus F)$ is disconnected. Compute the edge connectivity of G (by reducing to max flow computations). Try to use as few max flow computations as possible.
Hint: You will prove in CGT1 that the edge connectivity is equal to the size of the smallest edge cut, which is a set $F \subseteq E$ s.t. $(V, E \setminus F)$ is disconnected. Use this fact.

Goldberg and Unit Capacities: Analyze Goldberg's algorithm on networks with unit capacities. Will it be faster than other algorithms? Or at least as fast?

Goldberg with Highest Vertex: Design an implementation of improved Goldberg's algorithm where we always lift the highest vertex with a positive surplus. The number of unsaturated pushes will be $\mathcal{O}(n^2\sqrt{m})$ (you don't need to prove this), your goal is to design an algorithm matching this time.

Goldberg and Heights: What would happen, if we placed the source at height $n-1$, $n-2$ or even $n-3$? Figure out which property (it terminates, gives max flow always, etc.) depends on the height of the source, and what could go wrong.

Matrix Rounding: A matrix $A \in \mathbb{R}_{\geq 0}^{r \times s}$ is given, and you would like to round each of its entries up or down to an integer while preserving row and column sums. Design an algorithm which either gives a solution, or correctly declares the problem infeasible.