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5th tutorial

Dinic with Integers: How fast dies Dinic run when all capacities are 1? What if all capacities are bounded by some integer C? (express the complexity in terms of n, m and C.) Ideally we'd like to show that Dinic runs in these special cases at least as fast as Ford-Fulkerson.
Hint: Try to improve the bound on the computation of the blocking flow.

Vertex Disjoint Paths: Design an algorithm which finds the largest possible number of vertex disjoint paths between given two vertices $u, v \in V(G)$ in an undirected graph G.

Parliamentary Clubs: In a parliament with n members are m clubs. One MP may be a member of multiple clubs. Each club needs to elect among its members a chairman and a secretary. One MP cannot have two roles (e.g. be a chairman of two clubs, or be a chairman and secretary of two clubs, etc.). Design an algorithm which assigns roles (chair / secretary) to MPs, or reports that this is not possible.

Chessboard March: We are given an $n \times n$ chessboard with some cells inaccessible. The bottom row is occupied by pieces which can move one step ahead, or ahead left, or ahead right (i.e., north, north-west, or north-east). In one step, each piece is allowed to make at most one move (they are allowed to stay put), but one cell can contain at most one piece at any time. If a piece enters the top row, it disappears. Design an algorithm which finds the smallest number of steps such that it is possible to remove all pieces in this many steps, or reports, that no solution exists.

Min Vertex Cover: Design an algorithm computing the smallest *vertex cover* $C \subseteq V$ of a bipartite graph G. A subset C is a vertex cover if $\forall e \in E(G) \exists v \in C : v \in e$.

Vertex Connectivity: An undirected graph G has *vertex connectivity* k if removing any set of $k-1$ vertices does not disconnect the graph, but there exists a subset of vertices C of size k such that $G[V \setminus C]$ is disconnected. Compute the vertex connectivity of G (by reducing to max flow computations). Try to use as few max flow computations as possible.
Hint: You will prove in CGT1 that the vertex connectivity is equal to the size of the smallest vertex cut, which is a set $C \subseteq V$ s.t. $G[V \setminus C]$ is disconnected. Use this fact.