4th tutorial

1. #Paths in a DAG. Given a DAG G and its two vertices u, v, design an algorithm computing the number of distinct paths from u to v.

2. #Shortest Paths. Given a directed graph G and its two vertices u, v, design an algorithm computing the number of shortest paths from u to v.

3a. Parallel Planning. Imagine you are building a house or starting a business, and you have a *dependency graph.* This is a DAG $G = (V, E)$ whose vertices are tasks and an edge (u, v) means that task u has to be finished before task v can be started. Moreover, you have a length function $\ell : V \to \mathbb{N}$ which says that task u takes $\ell(u)$ time units to complete.
You are given a target date $T \in \mathbb{N}$, and you need to compute, for each task $u \in V$, what is the latest time $t(u)$ at which you can start executing this task so that everything is finished by time T. Assume you are able to do arbitrarily many tasks in parallel.

3b. Critical Vertices. Call a vertex u in G defined above *critical* if it corresponds to a task which, if it becomes delayed (equivalently, if $\ell(u)$ increases), the time the last task finishes also increases (i.e., the whole project is delayed). How to detect all critical vertices?

4. Computopia. The police department in the city of Computopia has made all streets one-way. The mayor contends that there is still a way to drive legally from any intersection in the city to any other intersection, but the opposition is not convinced. A computer program is needed to determine whether the mayor is right. However, the city elections are coming up soon, and there is just enough time to run a linear-time algorithm.

a) Formulate this problem graph-theoretically, and explain why it can indeed be solved in linear time.
b) Suppose it now turns out that the mayor's original claim is false. She next claims something weaker: if you start driving from town hall, navigating one-way streets, then no matter where you reach, there is always a way to drive legally back to the town hall. Formulate this weaker property as a graph-theoretic problem, and carefully show how it too can be checked in linear time