# 2nd tutorial



**1. Egg testing.** The Empire State Building is a 102-story skyscraper, and we know that if we throw an egg from the $K$-th floor or higher, it will break. We want to determine $K$, but use as few attempts (throws) as possible. What's the best strategy (minimizing the number of throws) if we have

*a*) one egg,
*b*) unlimited eggs,
*c*) two eggs,
*d*) (bonus: three eggs, or in general $e \in \mathbb{N}$ eggs?)

Is there anything special about the value 102?
(The egg can be a special one, so nothing can be *a priori* assumed about $K$.)

**2. Laser.** There is a row of $N$ buildings with $h_1, \ldots, h_n$ floors, and we need to demolish all of them. To that end, you found at home a demolition laser which is capable, in one firing, to either destroy an arbitrary building, or to destroy a given floor in all buildings (i.e., if you choose to destroy floor $L$, then the number of floors decreases by 1 for all buildings whose number of floors is $\geq L$). What is the minimum number of firings necessary to eliminate all buildings? (Beware that the maximum number of floors can be much more than $N$.)

**3. Representation.** We know three basic graph representations – list of edges, list of neighbors, and the adjacency matrix. Write out the complexity of the following standard operations:

- `add-edge(u,v)`
- `delete-edge(u,v)`
- `neighbors(u,v)`
- `degree(v)`
- `neighbor-list(v)`

*bonus:* What if in the "list of neighbors" representation you use a dictionary (implemented as a hash table) instead of a linked list? If you don't know what a dictionary is, skip the question.

**4. Exact Contiguous Subsequence.** You are given a sequence of *positive* numbers $x_1, \ldots, x_n$ and a number $K$. Find a contiguous subsequence whose sum is exactly $K$, i.e., some indices $i \leq j$ such that $x_i + x_{i+1} + \cdots + x_j = K$.

**5. Richest contiguous subsequence.** You are given a sequence of integers (possibly negative) $x_1, x_2, \ldots, x_n$. Find the richest contiguous subsequence, i.e. a subsequence $x_i, x_{i+1}, \ldots, x_j$ where $1 \leq i \leq j \leq n$ such that $\sum_{k=i}^{j} x_k$ is maximal. *bonus:* You are given a $m \times n$ matrix $M$. Find the richest contiguous submatrix (i.e., a rectangle) in $M$.

**6. Boss.** An adjacency matrix of a graph $G$ is preloaded into memory. Find a *boss* in $G$ or say that it doesn't contain any boss. A boss is a vertex who has an edge to all other vertices, and no edge is coming in to it.