koutecky@iuuk.mff.cuni.cz

4. tutorial

1. Lost Robots. There is a maze and 2 robots at different places in the maze. However, these robots are controlled by the same remote control, which has four buttons – going north, south, east, and west. When a robot receives a command which cannot be executed, it ignores it. How to find a sequence of commands which takes both robots out of the maze? (Once a robot is out of the maze, it stops listening to commands.) How to find the shortest such sequence?

2. Silly Robot. Again a grid maze with hallways and walls. In it, a robot which goes straight and only turns if it hits a wall or the boundary of the maze (i.e., reaches a cell adjacent to a wall, and reaches this cell from a direction orthogonal to the wall). You select the direction in which it turns (left, right, backwards). Find a path from a start cell s to a target cell t with the least number of turns.

3. Few flights. Let's have a map with vertices representing cities and "type 1" directed edges representing roads, and "type 2" edges representing flights. Assume that gas and car rental are cheap (haha), but flying is expensive, so you can afford at most k flights. Can the whole map be explored with just using k flights? (E.g., there could be cities which are not reachable only by road.)

---

4. Many paths. Construct a graph on n vertices with vertices s, t which has $2^{\Omega(n)}$ shortest paths from s to t.

5. Likeliest path. Imagine a computer network described by a directed graph, whose vertices correspond to routers and edges are links between them. For each link we have a probability of it being functional. The probability that a certain path is functional is the product of the probabilities of individual links along this path. How to find a path between two routers which is least likely to fail (most likely to work)?

6. Cheapest of shortest. Roads on a map are labeled with two numbers: length and toll. How to find the cheapest among the shortest paths?

7. Dijkstra fail. Construct a graph with integral lengths (without a negative cycle) where Dijkstra's algorithm would run exponentially long. (This is a variant of the algorithm which, if it relaxes a vertex v and changes the estimate for its neighbor u, it opens the neighbor, that is, puts it in the priority queue even if it extracted it previously.)

8. Negative edges $+k$? Can we get rid of negative edges by adding some number $k \in \mathbb{N}$ to all edge lengths, so that we can use Dijkstra's algorithm to find the shortest path even in a graph with negative edges?

9. Max-min tunnel. Let's have a map of a city represented by directed graph. Each edge is labeled by its clearance – what is the highest truck which can pass on this road? That is, given a path, the maximum height of a truck which can pass through the path is determined by the minimum clearance along the path. Given two vertices s, t, how to find a path which allows the tallest load to pass through (i.e., a path with the maximum minimum clearance)?